B.Sc Part I (Honrs) · Paper I

Solution of Symmetric functions of the Roots.

Q→ Find the numerical value of $(\alpha^2+2)(\beta^2+2)(\gamma^2+2)(\delta^2+2)$ where $\alpha, \beta, \gamma, \delta$ are the roots of the equation $x^4-7x^3+8x^2-5x+10=0$

Ans.→ Since $\alpha, \beta, \gamma$ and $\delta$ are the roots of the given equation then $x^4-7x^3+8x^2-5x+10=(x-\alpha)(x-\beta)(x-\gamma)(x-\delta)$ —(1)

Put $x=\sqrt{2}i$ we get

$$(\sqrt{2}i)^4-7(\sqrt{2}i)^3+8(\sqrt{2}i)^2-5(\sqrt{2}i)+10$$
$$=(\sqrt{2}i-\alpha)(\sqrt{2}i-\beta)(\sqrt{2}i-\gamma)(\sqrt{2}i-\delta)$$
$$\Rightarrow 4i^4-7\times 2\sqrt{2}i^3+8\times 2i^2-5\sqrt{2}i+10=(\alpha-\sqrt{2}i)(\beta-\sqrt{2}i)(\gamma-\sqrt{2}i)(\delta-\sqrt{2}i)$$
$$\Rightarrow 4+14+\sqrt{2}i-16-5\sqrt{2}i+10=(\alpha-\sqrt{2}i)(\beta-\sqrt{2}i)(\gamma-\sqrt{2}i)(\delta-\sqrt{2}i)$$
$$\Rightarrow -2+9\sqrt{2}i=(\alpha-\sqrt{2}i)(\beta-\sqrt{2}i)(\gamma-\sqrt{2}i)(\delta-\sqrt{2}i) \quad \text{—(2)}$$

In the same, We putting $x=-\sqrt{2}i$ in (1), we get

$$-2-9\sqrt{2}i=(\alpha+\sqrt{2}i)(\beta+\sqrt{2}i)(\gamma+\sqrt{2}i)(\delta+\sqrt{2}i) \quad \text{—(3)}$$

Multiplying (2) and (3), We get

$$(-2)^2-(9\sqrt{2}i)^2=(\alpha^2-2i^2)(\beta^2-2i^2)(\gamma^2-2i^2)(\delta^2-2i^2)$$
$$\Rightarrow 4-81\times 2i^2=(\alpha^2+2)(\beta^2+2)(\gamma^2+2)(\delta^2+2)$$
$$\Rightarrow 166=(\alpha^2+2)(\beta^2+2)(\gamma^2+2)(\delta^2+2)$$

Ans.

Q> If $\alpha, \beta, \gamma$ be the roots of the equation $ax^3+3bx^2+3cx+d=0$. Find the value in terms of

$$(2\alpha-\beta-\gamma)(2\beta-\gamma-\alpha)(2\gamma-\alpha-\beta).$$

Ans:- $\because \Sigma\alpha = -3b/a$

We have $2\alpha-\beta-\gamma = 3\alpha-(\alpha+\beta+\gamma) = 3\alpha-\Sigma\alpha$

$$= 3\alpha+\frac{3b}{a} = 3(\alpha+b/a)$$

In the same way, $2\beta-\gamma-\alpha = 3(\beta+b/a)$

and $2\gamma-\alpha-\beta = 3(\gamma+\frac{b}{a})$

from equation $(2\alpha-\beta-\gamma)(2\beta-\gamma-\alpha)(2\gamma-\alpha-\beta)$

$$= 3(\alpha+\frac{b}{a})\,3(\beta+\frac{b}{a})\,3(\gamma+b/a)$$

$$= 27(\alpha+\frac{b}{a})(\beta+\frac{b}{a})(\gamma+\frac{b}{a}) \text{ --- } (1)$$

We have $ax^3+3bx^2+3cx+d = a(x-\alpha)(x-\beta)(x-\gamma)$

Then one of the roots of $x$ in $-\frac{b}{a}$ as $\alpha+\frac{b}{a}=0 \Rightarrow$

$$\alpha = -b/a$$

$$\therefore a(-\frac{b}{a})^3+3b(-\frac{b}{a})^2+3c(-b/a)+d$$

$$= a(-\frac{b}{a}-\alpha)(-\frac{b}{a}-\beta)(-\frac{b}{a}-\gamma)$$

$$\Rightarrow -a\frac{b^3}{a^3}+3b\frac{b^2}{a^2}-\frac{3bc}{a}+d = -a(\frac{b}{a}+\alpha)(\frac{b}{a}+\beta)(\frac{b}{a}+\gamma)$$

$$\Rightarrow -a(\alpha+\frac{b}{a})(\beta+\frac{b}{a})(\gamma+\frac{b}{a})$$

$$= -\frac{b^3}{a^2}+\frac{3b^3}{a^2}-\frac{3bc}{a}+d$$

$$(\alpha+\frac{b}{a})(\beta+\frac{b}{a})(\gamma+\frac{b}{a}) = -\frac{2b^3-3abc+a^2d}{a^3}$$

$$= -(2b^3-3abc+a^2d)/a^3 \text{ --- } (2)$$

Using (2) in (1) we get

$$(2\alpha-\beta-\gamma)(2\beta-\gamma-\alpha)(2\gamma-\alpha-\beta) = 27\times\frac{-(2b^3-3abc+a^2d)}{a^3}$$

$$= -\frac{27}{a^3}(2b^3-3abc+a^2d) = \frac{27}{a^3}(3abc-2b^3-a^2d)$$ Ans.

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